

Preliminaries of probabilistic hierarchical fault detection

Ladislav Jirsa, Lenka Pavelková, and Kamil Dedecius

Department of Adaptive Systems,
Institute of Information Theory and Automation,
Academy of Sciences of the Czech Republic,
Pod Vodárenskou věží 4, 182 08 Prague, Czech Republic
{jirsa,pavelkov,dedecius}@utia.cas.cz
<http://www.utia.cas.cz>

Abstract. The paper proposes a novel probabilistic fault detection and isolation (FDI) system that enables to evaluate dynamically the industrial system condition (health) at any level of its functional hierarchy. The investigated industrial system is considered as a set of interconnected individual components. Each component acts in its noisy environment as an imperfect participant, more or less dependent on neighbouring components and, in turn, influencing some others. The nature of the problem prevents us from expressing sufficiently hard propositions about the health of the system as a whole at once but we can observe and construct propositions at lower system hierarchies. These propositions (opinions) are combined at higher levels using the rules of probabilistic logic, retaining the ignorance and finally yielding a single opinion on the health of the whole monitored system.

Keywords: Fault detection; FDI; probabilistic logic; system health.

1 Introduction

A fault is defined as an unpermitted deviation of at least one characteristic property of a variable from an acceptable behaviour. Therefore, the fault is a state that may lead to a malfunction or failure of the system [1, 2].

With increasing demands for safety and efficiency of complex processes, fault detection and isolation (FDI) becomes part of control systems in chemical, nuclear and aerospace engineering, automotive systems, power plant stations [3], software development etc. [4]. Together with FDI, controller capable to prevent failure or system reconfiguration that ensures the reliable and safe operation in the presence of component failures [5] might be implemented as well. FDI itself consists in binary opinion whether the system is in faulty state and indication of location and nature of the fault.

There exist three main classes of FDI methods: (i) *knowledge-based* FDI, exploiting human factor expertise, (ii) *signal-based* FDI considering properties of single or multiple signals and exploiting bounds checking, change-point detection, correlation and regression analyses etc., and (iii) *process-model-based* FDI,

reflecting a high-level model-based view on the whole manufacturing process. Quantitative methods use explicit process model in combination with statistics and decision theory, qualitative methods are based on artificial intelligence approach (pattern recognition, fuzzy theory, neural networks, spectral analysis etc.), see e.g. [6–9].

There are many process-model-based methods to evaluate faulty state, e.g. full-state observer-based methods or unknown input observer methods (using state-space system models), parity relations methods (using linear transformation of predicted and observed output), optimisation-based methods (minimising sensitivity to noise and maximising sensitivity to faults), methods based on Kalman filter, stochastic approach (description of a system by probabilistic distributions), system identification (tracking of model parameters), artificial intelligence techniques and others [4]. To deal with unobservable state variables, candidate methods are used to estimate the system evolution. A set of possible states (candidates) is constructed and used for comparison of output and the model to predict the expected future state of the system given each candidate [10], [11], [12]. The system structure can be represented either directly by the model or by using temporal logic describing possible sequence of events in case of fault occurrence within particular components [10],[12].

If we focus on an industrial plant, we may distinguish many possible fault sources. For instance, *sensors* are typically sources of gross errors, e.g. due to a fixed failure, a constant bias (positive or negative) or an out-of range failure. Some of the sensed variables are used for subsequent process control, which, under failure, may lead to significant degradation of production quality, unless this state is quickly detected and an appropriate action is taken. Another possible fault sources are *actuators*. While total breakdown of an actuator can be easily detected in most cases, a slow deterioration of actuator’s performance is a more challenging problem. Its detection can be easier if the actuator provides suitable feedback signal(s). *Hardware faults* stretch from trivial irreversible malfunction of a hardware component to hardly discoverable degradation of function caused e.g. by insufficient cooling of computer chip sets. *Software faults* (permanent or transient) may be caused by improper software configuration and incompatibilities, timing problems and even faults following from the lack of testing and bad programming habits. Other system faults can be initiated, e.g., by overloading of the operating system or communication lines due to exceptional situation, unacceptable signal-to-noise ratio, echoes etc.

The heterogeneous sources of faults inevitably place considerable demands on related FDI. The situation is yet more complicated due to different possible time developments of faults. In this respect, three basic fault types can be distinguished [1] and should be detectable by the proposed system: (i) *abrupt fault* causing undesirable stepwise change of a signal at once. This fault type is usually easily detected with only minimal delay. However, it may lead to immediate deviation of production quality beyond acceptable limits; (ii) *incipient fault* typical with its continuous drift from desirable value. Its recognition is closely tied with the character of the drift, mainly its time and “shape” properties; (iii) *intermit-*

tent fault occurring in intervals, usually irregular. These faults are generally very problematic due to their difficult detectability and isolation.

To summarize, a monitoring and processing of the system as a whole results generally in a solution (i) tailored for a particular system, i.e single-purpose (ii) combining different probability distributions of particular quantities of interest, either discrete or continuous, (iii) having a high dimensionality.

In this paper, we focus on a novel proposal of a dynamic FDI system based on probabilistic approach to fault detection. In the presented approach, the system of interest is decomposed into blocks, representing individual physical or logical system units (e.g. sensors, actuators, communication lines etc.). To each particular block, an observer is assigned that provides an opinion of the respective block health and related uncertainty. These observers can be considered as imperfect participants communicating via their connections within a structure. We aim to combine the information provided by involved participants to obtain a resulting value of system health.

The individual information pieces are fused together using the probabilistic logic framework in order to evaluate the health of the overall system. Probabilistic logic combines the capacity of probability theory to handle uncertainty with the capacity of deductive logic to exploit structure. We focus on a special type of probabilistic logic called subjective logic (SL) that explicitly takes uncertainty into account. It allows probability values to be expressed with degrees of uncertainty. In general, SL is suitable for modelling and analysing situations characterised by uncertainty and incomplete knowledge [13–15]. Note that the evaluation of opinions of the particular block health is not part of the current paper.

The paper is organised as follows. Section 2 provides basics of SL theory needed for its anticipated application to the problem of the system health monitoring. Section 3 gives a simple simulated example of industrial system and an evaluation of its health using rules of SL.

2 Representation and fusion of FDI-relevant knowledge

This section briefly deals with basics of SL framework as defined in [13–15]. We focus on such features that are important to the solving our problem that lies in (i) a representation of the knowledge about the health of individual industrial system blocks and (ii) combining these particular information pieces to obtain opinion of the overall health of the examined industrial system.

2.1 Basic notion of belief theory

In SL, the representation of uncertain probabilities is based on a belief model similar to the one used in [16]. The first step in applying this model is to define an exhaustive set of mutually exclusive elementary states of a given system, called the frame of discernment or state space and denoted by Θ . The powerset of Θ , denoted by 2^Θ , contains all possible subsets of Θ including Θ itself.

Elementary state in a frame of discernment Θ will be called atomic sets because they do not contain subsets. It is assumed that only one atomic set can be true at any one time. If a set is assumed to be true, then all supersets are considered true as well. An observer (subject, participant) who believes that one or several sets in the powerset of Θ might be true can assign belief masses to these sets. Belief mass on an atomic set $x \in 2^\Theta$ is interpreted as the belief that the set in question is true. Belief mass on a non-atomic set $x \in 2^\Theta$ is interpreted as the belief that one of the atomic sets it contains is true, but that the observer is uncertain about which of them is true.

A belief mass assignment (BMA) m_Θ distributes a total belief mass of 1 amongst the subsets of Θ such that the belief mass for each subset is positive or zero. Function $m_\Theta : 2^\Theta \rightarrow [0, 1]$ fulfills:

$$m_\Theta(x) \geq 0, \quad m_\Theta(\emptyset) = 0, \quad \sum_{x \in 2^\Theta} m_\Theta(x) = 1, \quad (1)$$

For each subset $x \in 2^\Theta$, the number $m_\Theta(x)$ is called the belief mass of x .

A belief mass $m_\Theta(x)$ expresses the belief assigned to the set x and does not express any belief in subsets of x in particular. A BMA is called dogmatic if $m_\Theta(\Theta) = 0$ because the total amount of belief mass has been committed. In contrast to belief mass, the belief in a set must be interpreted as an observers total belief that a particular set is true. A belief in x not only depends on belief mass assigned to x but also on belief mass assigned to subsets of x . Each subset $x \subseteq \Theta$ such that $m_\Theta(x) > 0$ is called a focal element of Θ . Note that in case all focal elements are elementary states then we speak about Bayesian BMA. A total belief that a particular state is true is expressed by the belief function $b : 2^\Theta \rightarrow [0, 1]$ defined by

$$b(x) = \sum_{\emptyset \neq y \subseteq x} m_\Theta(y), \quad x, y \in 2^\Theta.$$

Similarly to belief, a disbelief is interpreted as the total belief that a state is not true. Disbelief function corresponding with m_Θ is the function $d : 2^\Theta \rightarrow [0, 1]$ defined by

$$d(x) = \sum_{y \cap x = \emptyset} m_\Theta(y), \quad x, y \in 2^\Theta.$$

The uncertainty function corresponding with m_Θ is the function $u : 2^\Theta \rightarrow [0, 1]$ defined by

$$u(x) = \sum_{\substack{y \cap x \neq \emptyset \\ y \not\subseteq x}} m_\Theta(y), \quad x, y \in 2^\Theta.$$

Due to (1), the sum of the belief, disbelief and uncertainty functions is equal one, i.e.

$$b(x) + d(x) + u(x) = 1, \quad x \in 2^\Theta, \quad x \neq \emptyset. \quad (2)$$

In subjective logic, subjective opinions express specific types of beliefs, and represent the input and output arguments of the subjective logic operators.

Opinions expressed over binary state spaces are called binomial. Opinions defined over state spaces larger than binary are called multinomial. In this paper, we focus on the binomial opinion only as they suit FDI concept where the state space consist of two states, i.e. functionality/nonfunctionality of specific block of given system.

A MBA where the possible focal elements are Θ and/or elementary states (singletons) of Θ , is called a Dirichlet BMA function. The same mapping in the case of binary state spaces is called Beta belief mass distribution.

Base rate function $a : \Theta \rightarrow [0, 1]$ represents a priori probability expectation before any evidence has been received and fulfill

$$a(\emptyset) = 0 \text{ and } \sum_{x \in \Theta} a(x) = 1 \quad (3)$$

The combination of a Dirichlet MBA (or Beta MBA) and a base rate function can be comprised in a composite function called an opinion. Subjective opinions represent a special type of general belief functions. The subjective opinion model extend the traditional belief function model in the sense that opinions take base rates (it correspond to a prior information) into account whereas belief functions ignore base rates.

The probability transformation [17] projects a MBA onto a probability expectation value denoted by $p(x)$ as follows

$$p(x) = \sum_{y \subseteq \Theta} m_{\Theta}(y) \frac{|x \cap y|}{|y|}, \quad x, y \in 2^{\Theta} \quad (4)$$

2.2 Elements of binomial subjective opinions

A subjective opinion expresses a subjective belief of a particular subject (participant) about the truth of propositions including a degree of uncertainty. The propositions are represented by elementary states as defined in section 2.1. An opinion is denoted as ω_x^A where A is the subject who provides this opinion, and x is the proposition (state) to which the opinion applies. The proposition x is assumed to belong to a state space Θ which is usually not included in the opinion notation. The subject, the proposition and its frame are attributes of an opinion. Indication of subjective belief ownership is normally omitted whenever irrelevant, e.g. when only one subject is considered. A general multinomial opinion applies to a collection of propositions. A binomial opinion applies to a single proposition. Hereafter, we focus on binomial opinions only.

The binomial opinion is defined as follows: Let $\Theta = \{x, \bar{x}\}$ be a binary frame. A binomial opinion about the truth of state x is the ordered quadruple

$$\omega_x = (b, d, u, a) \quad (5)$$

where:

belief b is the belief mass in support of x being true,

disbelief d is the belief mass in support of x being false,
uncertainty u is the amount of uncommitted belief mass,
base rate a is the a priori probability in the absence of committed belief mass.

These components satisfy (2) and it holds $b, d, u, a \in [0, 1]$.

The probability expectation $p(x)$ (4) is defined by

$$p(x) = E_x = b + au \quad (6)$$

2.3 Binomial Beta opinion

Binomial opinion class has an equivalence mapping to Beta probability density function (pdf) under specific conditions. This mapping then gives subjective opinions a basis in notions from classical probability and statistics theory.

A general uncertain binomial opinion (i.e. with $u > 0$) corresponds to a Beta pdf denoted as $B(p|\alpha, \beta)$ where α and β are its two evidence parameters, $p = p(x)$ is defined by (6). Beta pdfs are expressed as

$$B(p|\alpha, \beta) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} p^{\alpha-1} (1-p)^{\beta-1}, 0 \leq p \leq 1, \alpha > 0, \beta > 0, \quad (7)$$

with the restriction that $p \neq 0$ if $\alpha < 1$ and $p \neq 1$ if $\beta < 1$.

Let r denote the number of observations of x , and let s denote the number of observations of \bar{x} . Then α and β parameters can be expressed as a function of the observations (r, s) in addition to the base rate a .

$$\alpha = r + Wa \quad (8)$$

$$\beta = s + W(1 - a)$$

The default non-informative prior weight $W = 2$ produces a uniform Beta pdf in case of default base rate $a = 1/2$ and $r = s = 0$.

The probability expectation value of the Beta pdf is defined as follows

$$E(B(p|\alpha, \beta)) = \frac{\alpha}{\alpha + \beta} = \frac{r + Wa}{r + s + W}. \quad (9)$$

The mapping from the parameters of a binomial opinion $\omega_X = (b, d, u, a)$ to the parameters of $B(p|\alpha, \beta)$ is defined as follows.

Let $\omega_X = (b, d, u, a)$ be a binomial opinion, and let $B(p|\alpha, \beta)$ with α, β defined by (8) be a Beta pdf, both over the same proposition x , i.e over the binary state space $\{x, \bar{x}\}$.

The opinions ω_X and $B(p|\alpha, \beta)$ are equivalent through the following mapping:

$$\left. \begin{array}{l} b = \frac{r}{W+r+s} = \frac{\alpha-Wa}{\alpha+\beta} \\ d = \frac{s}{W+r+s} = \frac{\beta-W(1-a)}{\alpha+\beta} \\ u = \frac{W}{W+r+s} = \frac{W}{\alpha+\beta} \end{array} \right\} \Leftrightarrow \left\{ \begin{array}{ll} \text{for } u \neq 0 : & \text{for } u = 0 : \\ \alpha = \frac{W(b+au)}{u} & E[p] = b \\ \beta = \frac{W(d+(1-a)u)}{u} & \\ 1 = b + d + u & 1 = b + d \end{array} \right. \quad (10)$$

The equivalence between binomial opinions and Beta pds is very powerful because subjective logic operators then can be applied to density functions and vice versa, and also because binomial opinions can be determined through statistical observations. For more details see [15].

2.4 Operators of subjective logic

Subjective logic provides a set of operators where input and output arguments are in the form of binomial opinions defined over binary frames. By using these operators, an efficient computation of mathematically complex models is enabled. Most of the operators correspond to well-known operators from binary logic and probability calculus. Additional operators exist for modelling special situations, such as when fusing opinions of multiple observers.

Below, some selected operators are described in detail:

Let $\Theta_1 = \{x, \bar{x}\}$ and $\Theta_2 = \{y, \bar{y}\}$ be two separate frames with independent opinions $\omega_X = (b_x, d_x, u_x, a_x)$ and $\omega_Y = (b_y, d_y, u_y, a_y)$, respectively.

Binomial multiplication corresponds to the logical AND and probability product. Notation: $\omega_{X \wedge Y} = \omega_X \cdot \omega_Y$

$$\begin{aligned} b_{x \wedge y} &= b_x b_y + \frac{(1 - a_x) a_y b_x u_y + (1 - a_y) a_x b_y u_x}{1 - a_x a_y} \\ d_{x \wedge y} &= d_x + d_y - d_x d_y \\ u_{x \wedge y} &= u_x u_y + \frac{(1 - a_y) b_x u_y + (1 - a_x) b_y u_x}{1 - a_x a_y} \\ a_{x \wedge y} &= a_x a_y \end{aligned} \quad (11)$$

Binomial comultiplication corresponds to the logical OR and probability coproduct. Notation: $\omega_{X \vee Y} = \omega_X \sqcup \omega_Y$

$$\begin{aligned} b_{x \vee y} &= b_x + b_y - b_x b_y \\ d_{x \vee y} &= d_x d_y + \frac{(1 - a_y) a_x d_x u_y + (1 - a_x) a_y d_y u_x}{a_x + a_y - a_x a_y} \\ u_{x \vee y} &= u_x u_y + \frac{a_y d_x u_y + a_x d_y u_x}{a_x + a_y - a_x a_y} \\ a_{x \vee y} &= a_x + a_y - a_x a_y \end{aligned} \quad (12)$$

Let two subjects A and B observe the same $X = \{x, \bar{x}\}$ in the same time instant and evaluate their opinions ω_X^A and ω_X^B .

Averaging Fusion $\omega_X^{A \diamond B} = \omega_X^A \oplus \omega_X^B$

$$\begin{aligned} b^{A \diamond B} &= \frac{b^A u^B + b^B u^A}{u^A + u^B} \\ u^{A \diamond B} &= \frac{2u^A u^B}{u^A + u^B} \end{aligned} \quad (13)$$

3 Example: monitoring of the system condition

In this section, an application of SL to the problem of the health monitoring of a technological process is presented.

We suppose that the investigated system is composed of a set of basic blocks. We assume that the blocks are monitored by a device-specific subjects that provide binary opinions on the functionality of the relevant blocks. This information is transformed into opinion on the block functionality using (10). We demonstrate the principle of combining involved opinion only. Also, the influence of changes in one block on the whole system is examined. For these purposes, we use a simulated system as defined below.

Let us consider a simple system of position adjustment to be monitored (see Fig. 1). The system consists of three basic blocks X (position measurement), Y (velocity measurement) and Z (actuator) that are organised in two units. The first unit contains blocks X and Y . They are interchangeable, i.e. information obtained by Y can be used to substitute information by X and vice versa (redundancy). The functionality of the sensor X is monitored by two subjects A (analysing noise and giving opinion ω_X^A) and B (analysing response and giving opinion ω_X^B), the functionality of the sensor Y is monitored by subject C (giving opinion ω_Y^C). The functionality of the actuator Z in the second unit is monitored by subject D (giving opinion ω_Z^D). Note that Fig. 1 does not describe physical composition of the system but units in a hierarchical structure showing how information of one unit affects the others. Each unit can be, on different levels of abstraction, created by sub-units etc.

Subjects A and B observe the same X simultaneously. Their opinions are therefore composed together by averaging fusion (13). The sensors X and Y are interchangeable, i.e. functionality of at least one of them is sufficient for a correct performance of the system. Therefore, opinions on their functionality is represented by comultiplication (12). Finally, the two major units must both work at the same time, therefore opinion on their mutual operation state is obtained as a multiplication (11) of opinions on each unit.

We denote subjects' opinions as ω_X^A , ω_X^B , ω_Y^C , ω_Z^D and opinion on the overall system functionality as ω . Then, using notation from Table ??, we get

$$\omega \equiv (b, d, u, a) = [(\omega_X^A \oplus \omega_X^B) \sqcup \omega_Y^C] \cdot \omega_Z^D. \quad (14)$$

The opinions are given as follows

$$\begin{aligned} \omega_X^A &= (0.9, 0.0, 0.1, 0.8) \\ \omega_X^B &= (0.8, 0.1, 0.1, 0.5) \\ \omega_Y^C &= (0.8, 0.2, 0.0, 0.5) \\ \omega_Z^D &= (0.9, 0.0, 0.1, 0.3) \end{aligned} \quad (15)$$

which indicate a high belief in the blocks' functionality with low (or absent) uncertainty and prior doubts about the block D ($a = 0.3$). Then, according to (14), $\omega = (0.89, 0.01, 0.1, 0.25)$ indicating high functionality of the system.

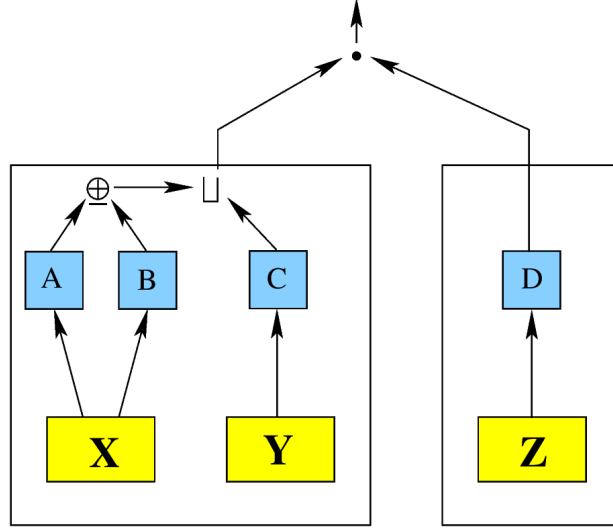


Fig. 1. Scheme of a monitored system. X, Y : monitored sensors, Z : another device, A, B, C, D : monitoring subjects

If we decrease belief in ω_X^A to $\omega_X^A = (0.2, 0.7, 0.1, 0.8)$, we get $\omega = (0.83, 0.08, 0.09, 0.25)$ which still keeps high performance because of the redundancy of X and Y .

If we use (15) but decrease belief in ω_Z^D to $\omega_Z^D = (0.2, 0.7, 0.1, 0.3)$, we get $\omega = (0.20, 0.70, 0.09, 0.75)$ showing poor overall performance and strong influence of the isolated block Z .

A set of experiment follows that examines how changes in the opinion of one block influence the behaviour of the whole system.

3.1 Influence of belief and disbelief

Let us keep values in (15) except of ω_Z^D . We consider $\omega_Z^D = (b_Z^D, d_Z^D, 0.1, 0.3) = (b_Z^D, 0.9 - b_Z^D, 0.1, 0.3)$ where b_Z^D lies within possible ranges given by (2), i.e. $b_Z^D \in [0, 0.9]$. The dependence of individual entries of ω on b_Z^D is shown in Fig. 2. The b and d are influenced very strongly because D enters the top level directly.

Now, we consider varying belief/disbelief in ω_X^A whereas other opinions fulfill (15). Similarly to the above mentioned case, $\omega_X^A = (b_X^A, 0.9 - b_X^A, 0.1, 0.8)$, where $b_X^A \in [0, 0.9]$. Results are shown in Fig. 3. It is obvious that an influence of the subject A involved in a more complex unit is less significant than in the previous case.

3.2 Influence of uncertainty

Now, we use (15) but change uncertainty in $\omega_Z^D, u_Z^D \in [0, 1]$. Then, according to (2), $b_Z^D = 1 - u_Z^D$ and $\omega_Z^D = (1 - u_Z^D, 0.0, u_Z^D, 0.3)$. The course of ω is shown in Fig. 4.

The same experiment for $\omega_X^A = (1 - u_X^A, 0, u_X^A, 0.8)$ is shown in Fig. 5, $u_X^A \in [0, 1]$.

Again, we can see a strong influence of ω_Z^D . In this experiment, influence of A 's uncertainty is practically negligible because the opinion on measurement is backed-up both by another subject B and also by another sensor Y .

3.3 Influence of base rates change

Let (15) be used and base rate a_Z^D is to be varying: $\omega_Z^D = (0.9, 0.0, 0.1, a_Z^D)$, $a_Z^D \in [0, 1]$. The influence of a_Z^D on overall ω is shown in Fig. 6. Increasing prior judgement a_Z^D on isolated block Z increases belief and decreases uncertainty of overall opinion, whereas disbelief remains practically unchanged. The most significant effect is linear increase of base rate, see (11).

Finally, we use (15) and vary $a_X^A \in [0, 1]$ in $\omega_X^A = (0.9, 0.0, 0.1, a_X^A)$. The influence on ω is shown in Fig. 7. Overall base rate is, again, affected most significantly. On the other hand, increasing value of a_X^A , increases uncertainty and slightly decreases disbelief.

4 Conclusion

In this paper, we proposed a novel type of probabilistic logic-based fault detection system with a highly modular and scalable structure. The decomposed system is represented by a collection of interconnected blocks, that can be interpreted as individual participants, whose opinions on particular block health is evaluated via Bayesian modelling. The methodology to obtain these opinions is specific according to nature of a particular unit and it is not addressed in the present paper. In order to obtain an information about the health of the whole monitored system, these particular opinions are fused together using the rules of probabilistic (more precisely subjective) logic. The resulting FDI system provides the human operator with information about the system functionality as a whole and at the same time enables to recognise health of particular blocks.

The proposed methodology (i) has capability of modular description and scalability, (ii) enables individual application of suitable probabilistic mechanism for each block, and (iii) avoids the dimensionality problem by using combination of low-dimensional units.

The future work comprises (i) evaluation of opinions on system health at each block and (ii) analysis of feasibility of the proposed system.

Acknowledgement

The project is supported by the grant MŠMT 7D12004 (E!7262 ProDiSMon) and by the Czech Science Foundation, project no. 13-13502S.

References

1. Isermann, R.: Model-based fault-detection and diagnosis status and applications. *Annual Reviews in Control* **29**(1) (2005) 71 – 85
2. Isermann, R.: *Fault Diagnosis Applications: Model Based Condition Monitoring, Actuators, Drives, Machinery, Plants, Sensors, and Fault-tolerant Systems*. Springer Verlag (2011)
3. Huang, X., Qi, H., Liu, X.: Implementation of fault detection and diagnosis system for control systems in thermal power plants. In: *Proceedings of the 6th World Congress on Intelligent Control and Automation, Dalian, China, Institute of Electrical and Electronics Engineers (IEEE) (June 21–23 2006)* 5777–5781
4. Hwang, I., Kim, S., Kim, Y., Seah, C.E.: A survey of fault detection, isolation, and reconfiguration methods. *IEEE Transactions on Control Systems Technology* **18**(3) (May 2010)
5. Zhang, Y., Jiang, J.: Bibliographical review on reconfigurable fault-tolerant control systems. *Annual Reviews in Control* **32**(2) (2008) 229 – 252
6. Ding, S.: *Model-based fault diagnosis techniques: design schemes, algorithms, and tools*. Springer (2008)
7. Gustafsson, F.: Statistical signal processing approaches to fault detection. *Annual Reviews in Control* **31**(1) (2007) 41–54
8. Venkatasubramanian, V., Rengaswamy, R., Yin, K., Kavuri, S.N.: A review of process fault detection and diagnosis: Part I: Quantitative model-based methods. *Computers & Chemical Engineering* **27**(3) (March 2003) 293–311
9. Venkatasubramanian, V., Rengaswamy, R., Kavuri, S.N., Yin, K.: A review of process fault detection and diagnosis: Part III: Process history based methods. *Computers & Chemical Engineering* **27**(3) (March 2003) 327–346
10. Williams, B.C., Nayak, P.P.: A model-based approach to reactive self-configuring systems. In: *In Proceedings of AAAI-96*. (1996) 971–978
11. Balaban, E., Cannon, H.N., Narasimhan, S., Brownston, L.S.: Model-based fault detection and diagnosis system for NASA Mars subsurface drill prototype. In: *2007 IEEE Aerospace Conference, Big Sky, Montana, Institute of Electrical and Electronics Engineers (IEEE) (March 2007)* 13
12. Magni, L., Scattolini, R., Rossi, C.: A fault detection and isolation method for complex industrial systems. *IEEE Transactions on Systems, Man and Cybernetics — Part A: Systems and Humans* **30**(6) (November 2000) 860–864
13. Jøsang, A.: A logic for uncertain probabilities. *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems* **9**(03) (2001) 279–311
14. Jøsang, A.: Probabilistic logic under uncertainty. In: *Proceedings of the thirteenth Australasian symposium on Theory of computing - Volume 65. CATS '07, Darlinghurst, Australia, Australia, Australian Computer Society, Inc. (2007)* 101–110
15. Jøsang, A.: Subjective logic. Draft book Available at: http://persons.unik.no/josang/papers/subjective_logic.pdf, visited **26** (2010)
16. Shafer, G.: *A mathematical theory of evidence*. Princeton University Press (1976)
17. Dubois, D., Prade, H.: On several representations of an uncertain body of evidence. *Fuzzy Information and Decision Processes* (1982) 167–181

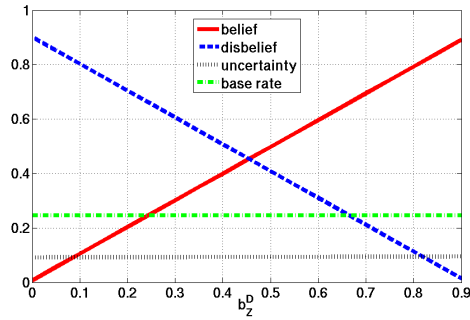


Fig. 2. Dependence of overall opinion $\omega = (b, d, u, a)$ on belief b_Z^D of subject D . Solid line: belief b , dashed line: disbelief d , dotted line: uncertainty u , dash-dot line: base rate a

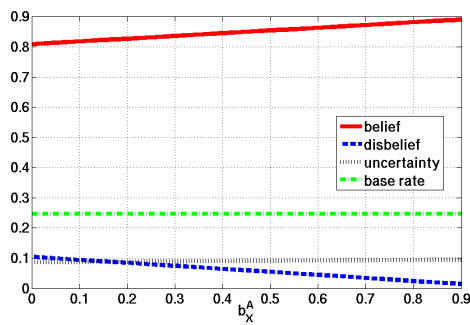


Fig. 3. Dependence of overall opinion $\omega = (b, d, u, a)$ on belief b_X^A of subject A . Solid line: belief b , dashed line: disbelief d , dotted line: uncertainty u , dash-dot line: base rate a

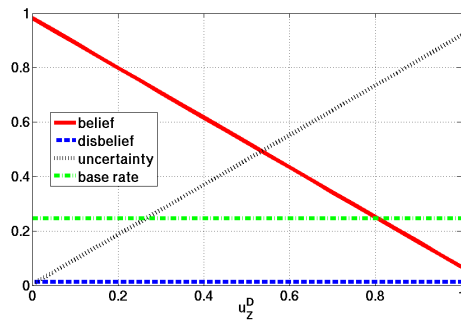


Fig. 4. Dependence of overall opinion $\omega = (b, d, u, a)$ on uncertainty u_z^D of subject D . Solid line: belief b , dashed line: disbelief d , dotted line: uncertainty u , dash-dot line: base rate a

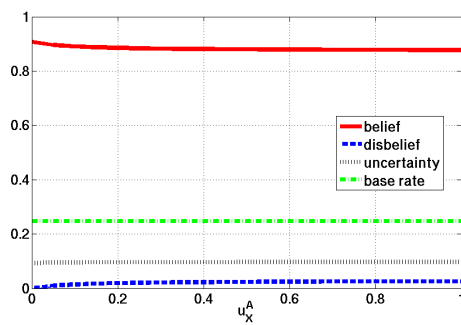


Fig. 5. Dependence of overall opinion $\omega = (b, d, u, a)$ on uncertainty u_x^A of subject A . Solid line: belief b , dashed line: disbelief d , dotted line: uncertainty u , dash-dot line: base rate a

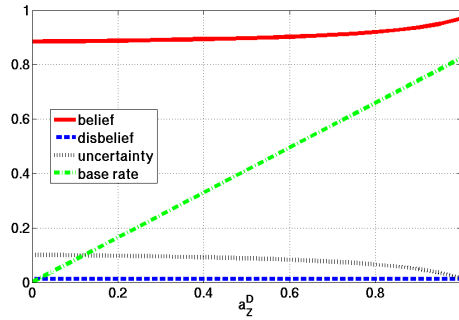


Fig. 6. Dependence of overall opinion $\omega = (b, d, u, a)$ on base rate a_Z^D of subject D . Solid line: belief b , dashed line: disbelief d , dotted line: uncertainty u , dash-dot line: base rate a

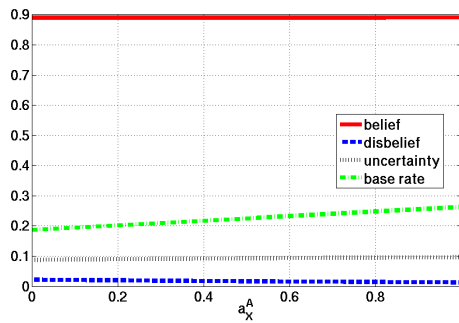


Fig. 7. Dependence of overall opinion $\omega = (b, d, u, a)$ on base rate a_X^A of subject A . Solid line: belief b , dashed line: disbelief d , dotted line: uncertainty u , dash-dot line: base rate a